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# Analytical solution for clay plug swelling experiments

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## Abstract

Clay swelling experiments frequently involve monitoring the one-dimensional displacement with time of an initially dry clay plug as it imbibes water from a supply at its base. This article presents a new analytical solution for interpreting such experiments based on Richards' equation for flow in a partially saturated porous medium combined with a linear empirical function relating moisture ratio with void ratio. The analytical solution is described by just two parameter groups. The first parameter group describes the swelling potential of the clay. The second parameter group describes the rate at which the swelling plug reaches equilibrium, which is controlled by permeability and capillary pressure. Application of the analytical solution is demonstrated by calibration to one-dimensional displacement data from clay swelling experiments for an illite and bentonite clay.

**Keywords:** Swelling, Unsaturated, Clay, Diffusion, Absorption, Richards' equation

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## 1. Introduction

Compacted Wyoming bentonite, a clay rock composed predominantly of sodium montmorillonite, is widely used as a barrier material for nuclear radioactive waste repositories, as an adsor-

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11 bent, and as an analogue for swelling clay mineral rich sub-sea shale formations (Komine & Ogata,  
12 1994). The two-dimensional layered structure of aluminosilicate clay minerals results in a high  
13 surface area and, combined with the presence of exchangeable cations, gives rise to the adsorbent  
14 properties of bentonite for radionuclides (for example,  $\text{Cs}^+$ ), and other contaminants (Eriksen et  
15 al., 1999).

16 These properties of bentonite and related clay minerals also result in a propensity to sponta-  
17 neously hydrate upon contact with water. To accommodate this hydration, the bulk volume of the  
18 clay mineral must increase if the material is unconfined; if confined, a swelling pressure arises  
19 instead. Upon swelling, the transport properties of the hydrated clay mineral material also change  
20 due to the changes in pore size distribution.

21 One manifestation of swelling that presents significant challenges to oil and gas operations  
22 is shale instability. Oil and gas reservoirs are usually topped by an impervious cap rock, which  
23 holds the less dense hydrocarbons within the subterranean reservoir. In most cases, the caprock  
24 will be a clay mineral rich shale. Thus shales are invariably encountered when drilling the well-  
25 bore to access the oil reservoir. To clear the rock fragments (cuttings) from the well-bore during  
26 drilling, and to maintain hydrostatic pressure in the cleared hole and thus formation integrity, a  
27 technical drilling fluid is used. This fluid is colloquially known as “mud”. In the early days  
28 of oil exploration, oil-based muds (OBM) were generally used, based on crude oil. However,  
29 owing to the environmental impacts of OBMs, drilling engineers have focussed on developing less  
30 environmentally damaging water-based muds (WBM).

31 The clay minerals present in shale may either disperse (for example kaolinite) or swell (for  
32 example montmorillonite) upon contact with WBMs, causing instability in the well bore region,

33 such as swelling/sloughing, in the medium-long term, or increased plasticity and the aggrega-  
34 tion of shale cuttings to the drill string (“bit balling”) in the short term (Anderson et al., 2010).  
35 The swelling of clay is strongly controlled by the composition of pore-water (Chai et al., 2014;  
36 Chen, 2016; Chavali et al., 2017; Oren and Akar, 2017). To address these problems, the industry  
37 has tried to increase the environmental performance of WBM s through use of more sophisticated  
38 water-based technologies including small organic molecules to inhibit clay mineral/shale swelling.  
39 Compacted bentonite is frequently used as a model in which to probe the efficacy of the swelling  
40 inhibitors.

41 The swelling properties of clay minerals have long been studied, in aqueous brines and in  
42 organic solvents (Anderson et al., 2010), though often at the single crystal level. Clay crystal  
43 swelling has been partitioned into both crystalline and osmotic swelling regimes (Anderson et al.,  
44 2010). Studies have primarily focussed on the equilibrium swelling of the clay minerals, with the  
45 expansion of the crystal at given conditions of temperature and salinity tested against classical  
46 colloid theories, e.g. DLVO theory (Smalley, 2006). Though the crystal chemistry and swelling  
47 potential of clay minerals are relatively well understood (Wilson & Wilson, 2014; Anderson et al.,  
48 2010), the link between clay mineral crystal swelling and shale stability, where many components  
49 influence the bulk swelling (Van Oort, 2003), is less clear, and it has even been argued that clay  
50 mineral swelling properties are not relevant in the oil field and that shale swelling may not occur  
51 when artefacts of experiments are carefully accounted for (Santarelli & Carminati, 1995).

52 It is inherently challenging to create compacted shale-like materials that replicate the proper-  
53 ties and behaviour of saturated shale under wellbore conditions. As such, compacted bentonite  
54 and other natural clays will remain in use as a proxy for shales to allow testing of drilling fluid

55 formations and additives and it is imperative to account for the artefacts introduced during produc-  
56 tion of compacted clay mineral shale analogues to be able to discern performance improvements  
57 in new drilling fluid technologies (Santarelli & Carminati, 1995). In particular, and especially to  
58 attempt to understand bit balling, the short term swelling rates of clay minerals need to be better  
59 understood.

60 To address this, a non-contact one-dimensional displacement meter has been developed and  
61 used to measure the short term swelling response of compacted bentonite tablets (Sellick et al.,  
62 2017). Similar experiments in this context are also reported by Chen (2016). Modelling of the  
63 swelling process can help reveal the relative importance of processes contributing to the compacted  
64 bentonite swelling. Whereas very accurate computational chemistry methods, such as atomistic  
65 molecular dynamics, have enabled swelling energetics to be probed at the clay mineral crystal  
66 scale (Suter et al., 2011), and more complex multi-scale methods have been developed (Suter  
67 et al., 2015), these are computationally very expensive. The objective of this study is to improve  
68 understanding about the kinetics of hydration processes through an analytical model, which allows  
69 rapid calculation of swelling curves to be tested against experimental data.

70 Our approach builds on modeling techniques previously developed to understand swelling and  
71 shrinkage of clay rich soils (Philip, 1969; Smiles and Raats, 2005; Su, 2010). Such models em-  
72 ploy Richards' equation to describe water movement through partially saturated porous media  
73 combined with empirical relationships between moisture ratio (volume of water / total volume)  
74 and void ratio (volume of voids / total volume) (Peng and Horn, 2007). The swelling of clay gives  
75 rise to moving boundary conditions. However, these can be dealt with by using a material coordi-  
76 nate system, associated with a theoretical dry clay mass, as opposed to a spatial coordinate system

77 (Philip, 1969; Su, 2010).

78 In the context of the swelling experiments described above, the resulting non-linear partial dif-  
79 ferential equation (PDE) have traditionally been solved numerically using finite differences (Kim,  
80 1999). Although a range of analytical and quasi-analytical solutions for this class of problem have  
81 been sought in the past (Raats, 2002), these have mostly been in the context of infiltration. In  
82 this article, a linear relationship between moisture ratio and void ratio is assumed such that the  
83 resulting PDE is linear and can be solved analytically. An exact analytical solution for the one-  
84 dimensional displacement of a compacted clay tablet as a function of time is then derived. Finally,  
85 the efficacy of the analytical solution is demonstrated by calibration to experimental data sets from  
86 two different clays.

## 87 **2. Mathematical model**

88 The approach used here was originally developed by Philip (1969) in the context of swelling  
89 soils, and has been frequently applied in the context of soil-deformation modeling (Kim, 1999;  
90 Smiles and Raats, 2005; Su, 2010). An explanation of how this approach can be used to derive an  
91 analytical solution for clay plug swelling experiments is described as follows.

92 The bulk volume of an unsaturated clay mass,  $V_b$  [ $L^3$ ], can be defined by

$$V_b = V_w + V_c + V_a \quad (1)$$

93 where  $V_w$  [ $L^3$ ],  $V_c$  [ $L^3$ ] and  $V_a$  [ $L^3$ ] are the volumes of water (including free and adsorbed water),  
94 clay mineral (excluding all water) and air present within the bulk clay mass of concern, respec-

95 tively.

96 The swelling of clay due to the presence of water manifests itself through a strong correlation  
97 between void ratio,  $e$  [-], and moisture ratio,  $\vartheta$  (e.g. Peng and Horn, 2007; Chertkov, 2012), defined  
98 by:

$$e = \frac{V_w + V_a}{V_c} \quad (2)$$

99

$$\vartheta = \frac{V_w}{V_c} \quad (3)$$

100 A simple way to represent this correlation is to assume that  $e$  and  $\vartheta$  are related by the linear  
101 function

$$e = e_I + (\vartheta_s - e_I) \left( \frac{\vartheta - \vartheta_I}{\vartheta_s - \vartheta_I} \right) \quad (4)$$

102 where  $e_I$  [-],  $\vartheta_I$  [-] and  $\vartheta_s$  [-] represent the initial void ratio, initial moisture ratio and maximum  
103 moisture ratio of a clay sample, respectively.

104 Consider water movement through the bulk clay associated with a mineral clay volume of  $\delta V_c$ .  
105 Let  $\delta Q$  [ $\text{L}^3\text{T}^{-1}$ ] be the net rate of out-flowing water. Assuming the water and mineral clay to be  
106 incompressible, the change in moisture ratio,  $\delta\vartheta$ , over a time-period,  $\delta t$  [T], is found from

$$\delta\vartheta\delta V_c = -\delta Q\delta t \quad (5)$$

107 such that for an infinitesimal time-period and mineral clay volume

$$\frac{\partial\vartheta}{\partial t} = -\frac{\partial Q}{\partial V_c} \quad (6)$$

## 2.1. One-dimensional vertical flow

Now consider a vertically oriented cylindrical clay plug encased within a rigid and impermeable sleeve. The clay is initially dried to a uniform moisture ratio,  $\vartheta_I$ , and uniform void ratio,  $e_I$ . The base of the clay plug is then soaked in water at a fixed pressure whilst the top of the plug is exposed to the atmosphere. In this way, the volumetric flow rate of water,  $Q$ , can be assumed to be described by the following form of Darcy's law (Philip, 1969)

$$Q = -\frac{Akk_r}{\mu_w} \frac{\partial(P_w + \rho_w gz + P_o)}{\partial z} \quad (7)$$

where  $A$  [ $L^2$ ] is the cross-sectional area of the clay plug (which is constant due to the rigid sleeve),  $k$  [ $L^2$ ] is the permeability,  $k_r$  [-] is relative permeability (which is a function of  $\vartheta$ ),  $P_w$  [ $ML^{-1}T^{-2}$ ] is water pressure,  $\mu_w$  [ $ML^{-1}T^{-1}$ ] is the dynamic viscosity of water,  $\rho_w$  [ $ML^{-3}$ ] is the density of water,  $g$  [ $LT^{-2}$ ] is gravitational acceleration,  $z$  [ $L$ ] is elevation and  $P_o$  [ $ML^{-1}T^{-2}$ ] is the so-called overburden pressure associated with the work done in swelling the clay. See Raats (2002) for further discussion with regards to the overburden pressure term.

Capillary pressure,  $P_c$  [ $ML^{-1}T^{-2}$ ], is defined by  $P_c = P_a - P_w$  where  $P_a$  [ $ML^{-1}T^{-2}$ ] is the air pressure. Importantly, capillary pressure is a function of  $\vartheta$ . Analogous to Richards' equation, assuming the air pressure to be constant throughout and capillary pressure variations to be much larger than those associated with gravity head and overburden pressure (which is very likely in clay media), Eq. (7) reduces to

$$Q = \frac{Akk_r}{\mu_w} \frac{\partial P_c}{\partial \vartheta} \frac{\partial \vartheta}{\partial z} \quad (8)$$



125 The relevant initial and boundary conditions are:

$$\begin{aligned}
 \vartheta &= \vartheta_I, \quad 0 \leq z \leq H_I, \quad t = 0 \\
 \vartheta &= \vartheta_s, \quad z = 0, \quad t > 0 \\
 Q &= 0, \quad z = H(t), \quad t > 0
 \end{aligned} \tag{9}$$

126 where  $H_I$  [L] is the initial height of the clay plug and  $H(t)$  [L] is the height of the clay plug at time,  
 127  $t$ . Note that the height of the clay plug increases with time as the clay plug swells due to water  
 128 adsorption within the clay layers.

## 129 2.2. Transformation to a static boundary condition

130 Considering Eqs. (1) and (2), the relationship between  $V_c$  and  $z$  is defined by

$$\frac{\partial V_c}{\partial z} = \frac{A}{e + 1} \tag{10}$$

131 Let us now consider an alternative coordinate,  $\zeta$  [L], defined by

$$\frac{\partial V_c}{\partial \zeta} = \frac{A}{e_I + 1} \tag{11}$$

132 It can be understood that

$$\frac{\partial \zeta}{\partial z} = \frac{e_I + 1}{e + 1} \tag{12}$$

133 and furthermore that

$$H(t) = \int_0^{H_I} \frac{e + 1}{e_I + 1} d\zeta \tag{13}$$

134 Eqs. (6), (8) and (9) map to the  $\zeta$  coordinate system to form a non-linear diffusion problem as  
 135 follows:

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial}{\partial \zeta} \left( D_A(\vartheta) \frac{\partial \vartheta}{\partial \zeta} \right) \quad (14)$$

136

$$\begin{aligned} \vartheta &= \vartheta_I, \quad 0 \leq \zeta \leq H_I, \quad t = 0 \\ \vartheta &= \vartheta_s, \quad \zeta = 0, \quad t > 0 \\ Q &= 0, \quad \zeta = H_I, \quad t > 0 \end{aligned} \quad (15)$$

137 where  $D_A$  [ $\text{L}^2\text{T}^{-1}$ ] is the apparent diffusion coefficient, defined by

$$D_A = - \frac{(e_I + 1)^2 k k_r}{(e + 1) \mu_w} \frac{\partial P_c}{\partial \vartheta} \quad (16)$$

138 The above set of equations can also be derived using the large deformation mathematical  
 139 framework of Song & Borja (2014).

### 140 2.3. Analytical solution for linear diffusion coefficient

141 For the special case when  $D_A$  can be treated as a positive constant, the above problem has an  
 142 analytical solution of the form (similar to Crank, 1975, p. 47)

$$\frac{\vartheta - \vartheta_I}{\vartheta_s - \vartheta_I} = 1 - 2 \sum_{n=0}^{\infty} \frac{1}{\lambda_n} \exp\left(-\frac{\lambda_n^2 D_A t}{H_I^2}\right) \sin\left(\frac{\lambda_n \zeta}{H_I}\right) \quad (17)$$

143 where  $\lambda_n = (n + 1/2)\pi$ .

Substituting Eq. (17) into Eq. (4) and then into Eq. (13) leads to

$$h(t) = \alpha \left[ 1 - 2 \sum_{n=0}^{\infty} \frac{1}{\lambda_n^2} \exp(-\beta \lambda_n^2 t) \right] \quad (18)$$

where

$$h(t) = \frac{H - H_I}{H_I}, \quad \alpha = \frac{\vartheta_s - e_I}{e_I + 1}, \quad \beta = - \left( \frac{e_I + 1}{H_I} \right)^2 \frac{kk_r}{(e + 1)\mu_w} \frac{\partial P_c}{\partial \vartheta} \quad (19)$$

and  $h(t)$  [-] is hereafter referred to as the swelling ratio. Note that this is the first and only time that the linear relationship between  $e$  and  $\vartheta$  (Eq. (4)) is invoked.

For a zero  $e_I$ ,  $\alpha = \vartheta_s$ . The  $\alpha$  parameter says something about the adsorption capacity of the clay. Higher values of  $\alpha$  imply greater adsorption potential and hence greater swelling potential. The  $\beta$  parameter represents the rate at which a clay swelling experiment equilibrates. Large values of  $\beta$  correspond to high permeability and/or high capillary pressures. Large  $\beta$  values lead to the clay reaching equilibrium faster.

### 3. Calibration to experimental data

Artificial cylindrical plugs of clay were processed with a thickness of 3.8 mm and a diameter of 20 mm. The clay plugs were placed into circular sample holders with the base exposed to a water bath. Clay plugs within the sample holders sat on top of circular perforated disks to enable hydraulic contact between the water bath and the base of the plugs. The sample holder was also designed to constrain the plug laterally such that swelling only occurs in the vertical direction. Vertical displacement of the upper surface of the clay plug was measured using an induction sensor non-contact one-dimensional displacement meter. The displacement of the upper

surface in the course of time was monitored. Two clay plugs were examined comprised of illite and bentonite. Using illite and bentonite (predominantly sodium montmorillonite) provide near end members in terms of clay swelling response in terms of crystalline swelling. Assuming similar particle size/void space, the additional swelling of any other minerals should fall between or close to these materials.

Values of  $\alpha$  [-] and  $\beta$  [ $T^{-1}$ ] were obtained by calibrating Eq. (18) to the observed data. Calibration was achieved by minimizing the mean absolute error (MAE) between observed and simulated  $h$  using MATLAB's local optimization routine, FMINSEACH. A comparison of observed and calibrated model results are presented in Fig. 1. The associated calibrated parameters and MAE are presented in Table 1. It is clear that the two parameter analytical solution is capable of catching almost all aspects of the data presented

Consider again Eq. (19). The fact that  $\alpha$  is larger for bentonite than for illite suggests that bentonite has a larger  $\vartheta_s$  and hence a greater potential for water adsorption. The fact that  $\beta$  is larger for illite as compared to bentonite suggests that illite has a higher permeability and/or a higher capillary pressure, leading to faster capillary suction of water.

#### 4. Conclusions

The objective of this study was to develop an analytical solution to aid interpretation of one-dimensional displacement observations from clay swelling experiments. An appropriate mathematical model was developed by combining Richards' equation for flow in partially saturated porous media with a linear empirical function relating void ratio with moisture ratio. By ignoring gravity and overburden pressure and assuming a constant relative permeability and moisture ratio

Table 1: Model calibration results.

Clay type	$\alpha$ (%)	$\beta$ (hour <sup>-1</sup> )	MAE (%)
Illite	44.4	10.1	0.747
Bentonite	70.1	1.06	0.794

- capillary pressure ratio, the associated non-linear partial differential equation reduced to a linear diffusion equation for the water ratio. Making use of Crank (1975), an analytical Fourier series solution for the water ratio in the swelling clay plug was found. This solution was substituted into the linear void ratio–moisture ratio function to obtain an expression for the void ratio, which in turn was used to calculate the height of the sample in the course of time.

The analytical solution reveals that one-dimensional displacement is controlled by two important parameter groups,  $\alpha$  and  $\beta$  (recall Eq. (19)). The  $\alpha$  parameter is linked to the maximum moisture ratio for the clay. Large  $\alpha$  values imply higher potential for water adsorption and swelling. The  $\beta$  parameter is linked to permeability and capillary pressure and represents the rate at which a clay swelling experiment reaches equilibrium.

The analytical solution was successfully calibrated to experimental data for two very different clays: illite and bentonite. The illite was found to have a lower swelling potential (low  $\alpha$ ) but was found to equilibrate relatively quickly (high  $\beta$ ). In contrast, the bentonite was found to have a much greater swelling potential (high  $\alpha$ ). However, swelling in the bentonite was found to develop more slowly (low  $\beta$ ).

Anticipated future work will look at empirical relationships between  $\alpha$  and  $\beta$  and various drilling fluid compositional characteristics, with a view that these can be used to simulate drilling fluid improvements on oil-reservoir scale models of clay swelling phenomena.

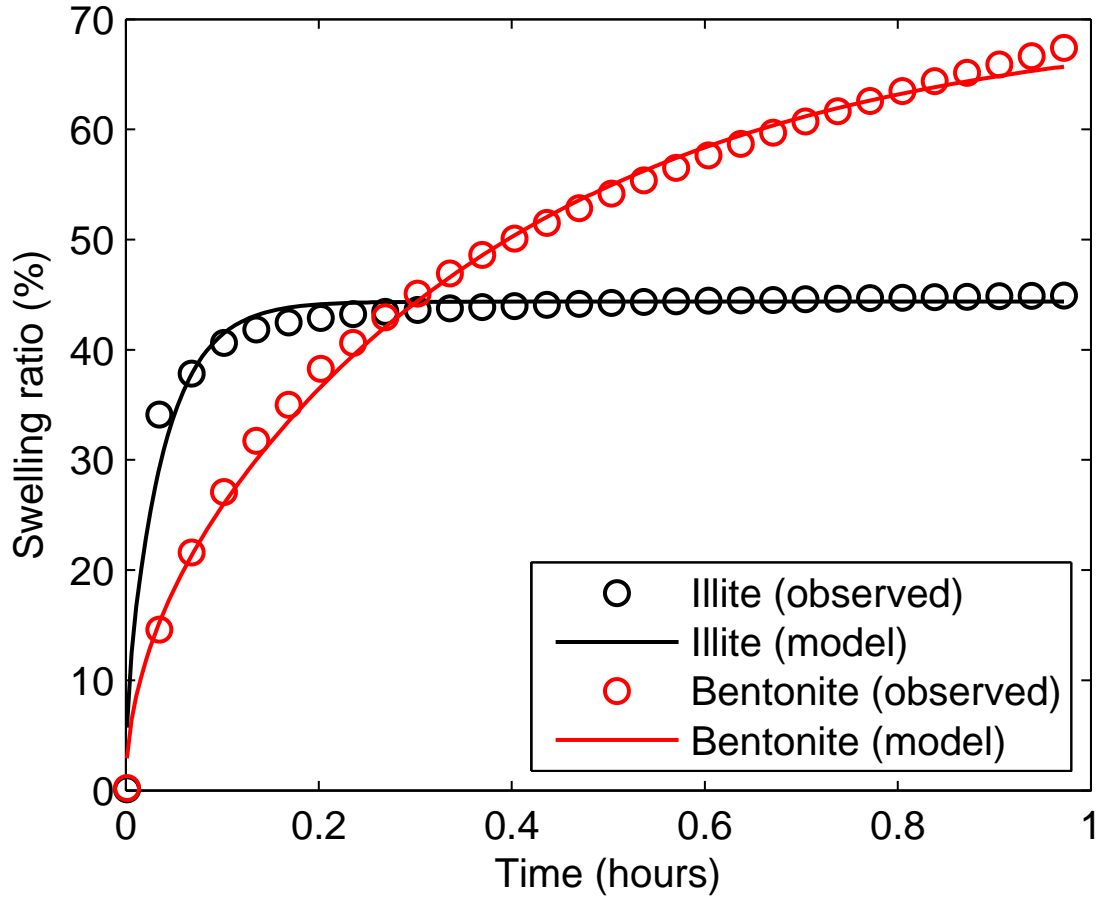


Figure 1: Plots of swelling ratio against time as observed during the experiments and as simulated from the calibrated models, according to Eq. (18).

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